Section 2.2, Exercise 75

A taxi-cab company charges \$ 2.00 for the first mile (or part of a mile) and \$ 0.20 for every tenth (or part of a tenth) of a mile beyond the first. Write a function $C(x)$ the computes the cost (in dollars) for a trip of distance x (in miles). Graph $C(x)$ for the domain $0 \leq x \leq 2$.

Solution: It helps to first computer some sample values for this function $C(x)$. For example, a 2-mile trip should cost \$ 4.00, since it will cost \$ 2.00 for the first mile, and then $10 \times \$0.20 = \$2.00$ for the second mile (the multiplication by 10 is done because 1 mile can be thought of as 10 tenths of a mile). Thus $C(2) = 4$. A $\frac{1}{2}$-mile trip should cost \$ 2.00, the same as any other trip that is a mile or less. A 1.1-mile trip should cost \$ 2.00 + \$ 0.20 - \$ 2.20, so $C(1.1) = 2.2$. A 1.05-mile trip, though should cost the same as a 1.1-mile trip since a part of a tenth is counted as a whole tenth. Thus $C(1.05) = 2.2$.

We could now construct a table to describe the situation better:

Distance (x)	Fare ($C(x)$)
$x = 0$	0
$0 < x \leq 1$	\$2.00
$1 < x \leq 1.1$	\$2.20
$1.1 < x \leq 1.2$	\$2.40
$1.2 < x \leq 1.3$	\$2.60
$1.3 < x \leq 1.4$	\$2.80
$1.4 < x \leq 1.5$	\$3.00
$1.5 < x \leq 1.6$	\$3.20
$\vdots$	$\vdots$

We can now construct $C(x)$. We will need to use the greatest integer function, $[\![x]\!]$. It essentially says "if x is positive round down, and if x is negative round up." It's described in section 2.2 if you want to know more.

We need to determine how much beyond 1 mile the taxi has travelled. That is described in miles by $(x - 1)$. However, we want to know how many *tenths* of a mile beyond 1 mile the taxi travels, so that is described by $10(x-1)$. To round this to whole tenths (which is how the fare is calculated), we use $[\![10(x-1)]\!]$. Finally we may multiply by how much is charged per tenth (or part of a tenth), getting $.2 \cdot [\![10(x-1)]\!]$. Now we may combine this with the initial \$2.00 charged for the first mile to get:

$$C(x) = \begin{cases} 0 & x = 0 \\ 2 & 0 < x \leq 1 \\ 2 + .2 \cdot [\![10(x-1)]\!] & x > 1 \end{cases}$$

The graph for $0 \leq x \leq 2$ should look like: