

7.5 5, 6, 15, 16, 35, 39, 44, 49, 54, 55, 56, 67, 77, 78

5. $2\sin x + \sqrt{3} = 0$

$$2\sin x = -\sqrt{3}$$

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$x = \frac{-\pi}{3} + 2k\pi, \frac{-2\pi}{3} + 2k\pi \text{ for any integer } k.$$

(Why not just $-\frac{\pi}{3}$ and $-\frac{2\pi}{3}$? Because $\sin x$ repeats itself every 2π (the period), each value of the form $-\frac{\pi}{3} + 2k\pi$ or $-\frac{2\pi}{3} + 2k\pi$ is also a solution. For example, $-\frac{\pi}{3} + 2\pi = \frac{5\pi}{3}$ and $-\frac{2\pi}{3} + 4\pi = \frac{10\pi}{3}$ are also solutions.)

6. $\tan x + 1 = 0$

$$\tan x = -1$$

$$x = \frac{3\pi}{4} + k\pi$$

To get this answer, I found all solutions between 0 and π (the period) and then added the $k\pi$ to get all solutions on all of \mathbb{R} .

15. $\cos x \sin x - 2\cos x = 0$

$$\cos x (\sin x - 2) = 0$$

[Factor]

$$\cos x = 0 \text{ or } \sin x - 2 = 0$$

$$\cos x = 0 \text{ when } x = \frac{\pi}{2} + 2k\pi \text{ or } \frac{3\pi}{2} + 2k\pi \text{ (more succinctly, } x = \frac{\pi}{2} + k\pi)$$

$$\sin x - 2 = 0 \text{ when } \sin x = 2 \text{ which is impossible since } \sin x \leq 1 \text{ for any } x.$$

$$\text{Thus the solutions are } x = \frac{\pi}{2} + k\pi.$$

16. $\tan x \sin x + \sin x = 0$

$$\sin x (\tan x + 1) = 0$$

[Factor]

$$\text{So } \sin x = 0 \text{ or } \tan x + 1 = 0.$$

$$\sin x = 0 \text{ when } x = 0 + 2k\pi \text{ or } \pi + 2k\pi, \text{ so } x = k\pi$$

$$\tan x + 1 = 0 \text{ when } x = \frac{3\pi}{4} + k\pi \text{ (see problem 6)}$$

$$\text{Thus the solutions are } x = k\pi \text{ or } \frac{3\pi}{4} + k\pi.$$

$$35. \quad 4\sin x \cos x + 2\sin x - 2\cos x - 1 = 0$$

$$2\sin x (2\cos x + 1) - (2\cos x + 1) = 0 \quad \text{[Factor]}$$

$$(2\sin x - 1)(2\cos x + 1) = 0 \quad \text{[Factor]}$$

$$\text{So } 2\sin x - 1 = 0 \text{ or } 2\cos x + 1 = 0$$

$$2\sin x - 1 = 0 \text{ when } \sin x = \frac{1}{2}, \text{ so } x = \frac{\pi}{6} + 2k\pi \text{ or } \frac{5\pi}{6} + 2k\pi$$

$$2\cos x + 1 = 0 \text{ when } \cos x = -\frac{1}{2}, \text{ so } x = \frac{2\pi}{3} + 2k\pi \text{ or } \frac{4\pi}{3} + 2k\pi.$$

Thus the solutions are:

$$\frac{\pi}{6} + 2k\pi, \frac{2\pi}{3} + 2k\pi, \frac{5\pi}{6} + 2k\pi, \text{ and } \frac{4\pi}{3} + 2k\pi. \quad \text{for } k \text{ any integer.}$$

$$39. \quad 2\cos 3x = 1$$

$$\cos(3x) = \frac{1}{2}$$

$$\text{So } 3x = \frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$$

$$\text{Thus } x = \frac{\pi}{9} + \frac{2}{3}k\pi, \frac{5\pi}{9} + \frac{2}{3}k\pi$$

$$\text{So the solutions are } x = \frac{\pi}{9}, \frac{\pi}{9} + \frac{2\pi}{3} = \frac{7\pi}{9}, \frac{\pi}{9} + \frac{4\pi}{3} = \frac{13\pi}{9}, \frac{\pi}{9} + \frac{6\pi}{3} = \frac{19\pi}{9}, \dots$$

$$\text{and } x = \frac{5\pi}{9} - \frac{2\pi}{3} = \frac{-\pi}{9}, \frac{5\pi}{9}, \frac{5\pi}{9} + \frac{2\pi}{3} = \frac{11\pi}{9}, \frac{5\pi}{9} + \frac{4\pi}{3} = \frac{17\pi}{9}, \frac{5\pi}{9} + \frac{6\pi}{3} = \frac{23\pi}{9}, \dots$$

Thus the solutions in the range $0 \leq x < 2\pi$ are:

$$x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$$

$$44. \quad 2\sin^2 x - \cos x = 1.$$

$$2(1 - \cos^2 x) - \cos x = 1 \quad \text{[Substitute } \sin^2 x = 1 - \cos^2 x \text{]}$$

$$2 - 2\cos^2 x - \cos x = 1$$

$$-2\cos^2 x - \cos x + 1 = 0$$

Since we can factor $-2C^2 - C + 1 = (-2C + 1)(C + 1)$, we can factor the above

$$(-2\cos x + 1)(\cos x + 1) = 0$$

$$\text{Thus } -2\cos x + 1 = 0 \text{ or } \cos x + 1 = 0$$

$$\text{so } \cos x = \frac{1}{2} \text{ or } \cos x = -1.$$

Thus the solutions in the range $0 \leq x < 2\pi$ will be

$$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}.$$

49 $\sec x - 5 = 0$

$\sec x = 5$

$\frac{1}{\cos x} = 5$

$\cos x = \frac{1}{5}$

a) $x = \cos^{-1}\left(\frac{1}{5}\right) + 2\pi k$ and $-\cos^{-1}\left(\frac{1}{5}\right) + 2\pi k$

b) $\cos^{-1}\left(\frac{1}{5}\right) \approx 1.36944$

$-\cos^{-1}\left(\frac{1}{5}\right) + 2\pi \approx 4.91375$

54. $\tan^4 x - 13\tan^2 x + 36 = 0$

We can factor $T^2 - 13T + 36 = (T-4)(T-9)$, so we can get

$(\tan^2 x - 4)(\tan^2 x - 9) = 0$

So $\tan^2 x - 4 = 0$ or $\tan^2 x - 9 = 0$.

Thus $\tan x = \pm 2$ or $\tan x = \pm 3$, so

a) $x = \tan^{-1} 2 + \pi k$, $\tan^{-1}(-2) + \pi k$, $\tan^{-1} 3 + \pi k$, or $\tan^{-1}(-3) + \pi k$

b) $\tan^{-1} 2 \approx 1.10715$

$\tan^{-1} 3 \approx 1.24905$

$\tan^{-1}(2) + \pi \approx 4.24874$

$\tan^{-1} 3 + \pi \approx 4.39064$

$\tan^{-1}(-2) + \pi \approx 2.03444$

$\tan^{-1}(3) + \pi \approx 1.89255$

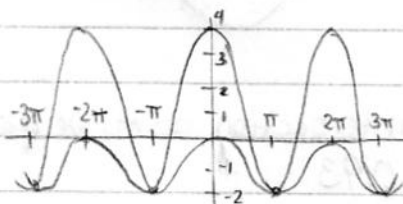
$\tan^{-1}(-2) + 2\pi \approx 5.17604$

$\tan^{-1}(-3) + 2\pi \approx 5.03414$

These are all solutions in the interval $[0, 2\pi)$.

55. $f(x) = 3\cos x + 1$

$g(x) = \cos x - 1$

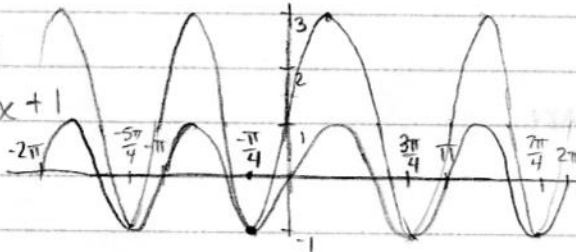


Points of intersection

 $((2k-1)\pi, -2)$ for k integerodd multiples of π

56. $f(x) = \sin 2x$

$g(x) = 2\sin 2x + 1$



Points of intersection:

 $(-\frac{\pi}{4} + k\pi, -1)$ for integer k .

67. $\sin 2x + \cos x = 0$ (Solve on $[0, 2\pi)$)

$$2\sin x \cos x + \cos x = 0$$

$$\cos x (2\sin x + 1) = 0$$

so $\cos x = 0$ or $2\sin x + 1 = 0$

$$\cos x = 0 \text{ when } x = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

$$2\sin x + 1 = 0 \text{ when } \sin x = -\frac{1}{2}. \text{ Thus } x = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}$$

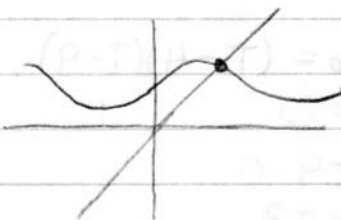
Hence the solutions are

$$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \text{ and } \frac{11\pi}{6}$$

77. $2^{\sin x} = x$

$$Y_1 = 2^{\sin(x)}$$

$$Y_2 = x$$



Use ZTrig for zoom

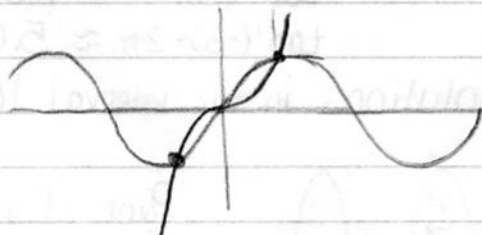
intersect gives us: $(1.9187, 1.9187)$

So the solution is $x \approx 1.92$

78. $\sin x = x^3$

$$Y_1 = \sin(x)$$

$$Y_2 = x^3$$



Using intersect to get each intersection point, we get the solutions

$$x = -0.93 \text{ and } x = 0.93$$

END OF 7.5

END OF LAST ASSIGNMENT - HOORAY!