

7.5 5, 6, 15, 16, 35, 39, 44, 49, 54, 55, 56, 67, 77, 78

5. $2\sin x + \sqrt{3} = 0$

$$2\sin x = -\sqrt{3}$$

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$x = \frac{-\pi}{3} + 2k\pi, \frac{-2\pi}{3} + 2k\pi \text{ for any integer } k.$$

(Why not just $-\frac{\pi}{3}$ and $-\frac{2\pi}{3}$? Because $\sin x$ repeats itself every 2π (the period), each value of the form $-\frac{\pi}{3} + 2k\pi$ or $-\frac{2\pi}{3} + 2k\pi$ is also a solution. For example, $-\frac{\pi}{3} + 2\pi = \frac{5\pi}{3}$ and $-\frac{2\pi}{3} + 4\pi = \frac{10\pi}{3}$ are also solutions.)

6. $\tan x + 1 = 0$

$$\tan x = -1$$

$$x = \frac{3\pi}{4} + k\pi$$

To get this answer, I found all solutions between 0 and π (the period) and then added the $k\pi$ to get all solutions on all of \mathbb{R} .

15. $\cos x \sin x - 2\cos x = 0$

$$\cos x (\sin x - 2) = 0$$

[Factor]

$$\cos x = 0 \text{ or } \sin x - 2 = 0$$

$$\cos x = 0 \text{ when } x = \frac{\pi}{2} + 2k\pi \text{ or } \frac{3\pi}{2} + 2k\pi \text{ (more succinctly, } x = \frac{\pi}{2} + k\pi)$$

$\sin x - 2 = 0$ when $\sin x = 2$ which is impossible since $\sin x \leq 1$ for any x .

Thus the solutions are $x = \frac{\pi}{2} + k\pi$.

16. $\tan x \sin x + \sin x = 0$

$$\sin x (\tan x + 1) = 0$$

[Factor]

So $\sin x = 0$ or $\tan x + 1 = 0$.

$\sin x = 0$ when $x = 0 + 2k\pi$ or $\pi + 2k\pi$, so $x = k\pi$

$\tan x + 1 = 0$ when $x = \frac{3\pi}{4} + k\pi$ (see problem 6)

Thus the solutions are $x = k\pi$ or $\frac{3\pi}{4} + k\pi$.

$$35. 4\sin x \cos x + 2\sin x - 2\cos x - 1 = 0$$

$$2\sin x (2\cos x + 1) - (2\cos x + 1) = 0 \quad \text{[Factor]}$$

$$(2\sin x - 1)(2\cos x + 1) = 0 \quad \text{[Factor]}$$

$$\text{So } 2\sin x - 1 = 0 \text{ or } 2\cos x + 1 = 0$$

$$2\sin x - 1 = 0 \text{ when } \sin x = \frac{1}{2}, \text{ so } x = \frac{\pi}{6} + 2k\pi \text{ or } \frac{5\pi}{6} + 2k\pi$$

$$2\cos x + 1 = 0 \text{ when } \cos x = -\frac{1}{2}, \text{ so } x = \frac{2\pi}{3} + 2k\pi \text{ or } \frac{4\pi}{3} + 2k\pi.$$

Thus the solutions are:

$$\frac{\pi}{6} + 2k\pi, \frac{2\pi}{3} + 2k\pi, \frac{5\pi}{6} + 2k\pi, \text{ and } \frac{4\pi}{3} + 2k\pi. \text{ for } k \text{ any integer.}$$

$$39. 2\cos 3x = 1$$

$$\cos(3x) = \frac{1}{2}$$

$$\text{So } 3x = \frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$$

$$\text{Thus } x = \frac{\pi}{9} + \frac{2}{3}k\pi, \frac{5\pi}{9} + \frac{2}{3}k\pi$$

$$\text{So the solutions are } x = \frac{\pi}{9}, \frac{\pi}{9} + \frac{2\pi}{3} = \frac{7\pi}{9}, \frac{\pi}{9} + \frac{4\pi}{3} = \frac{13\pi}{9}, \frac{\pi}{9} + \frac{6\pi}{3} = \frac{19\pi}{9}, \dots$$

$$\text{and } x = \frac{5\pi}{9} - \frac{2\pi}{3} = \frac{-\pi}{9}, \frac{5\pi}{9}, \frac{5\pi}{9} + \frac{2\pi}{3} = \frac{11\pi}{9}, \frac{5\pi}{9} + \frac{4\pi}{3} = \frac{17\pi}{9}, \frac{5\pi}{9} + \frac{6\pi}{3} = \frac{23\pi}{9}, \dots$$

Thus the solutions in the range $0 \leq x < 2\pi$ are:

$$x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$$

$$44. 2\sin^2 x - \cos x = 1.$$

$$2(1 - \cos^2 x) - \cos x = 1 \quad \text{[Substitute } \sin^2 x = 1 - \cos^2 x \text{]}$$

$$2 - 2\cos^2 x - \cos x = 1$$

$$-2\cos^2 x - \cos x + 1 = 0$$

Since we can factor $-2C^2 - C + 1 = (-2C + 1)(C + 1)$, we can factor the above

$$(-2\cos x + 1)(\cos x + 1) = 0$$

$$\text{Thus } -2\cos x + 1 = 0 \text{ or } \cos x + 1 = 0$$

$$\text{so } \cos x = \frac{1}{2} \text{ or } \cos x = -1.$$

Thus the solutions in the range $0 \leq x < 2\pi$ will be

$$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}.$$

