7.3 4, 11, 16, 23, 28, 33, 35.

4. $\csc x = 4$, $\tan x < 0$

Since $\csc x = 4$, we get $\sin x = \frac{1}{\csc x} = \frac{1}{4}$

Since $\tan x < 0$ and $\sin x > 0$, we know $\cos x < 0$.

Since $\cos^2 x = 1 - \sin^2 x$, we get $\cos^2 x = 1 - (\frac{1}{4})^2 = \frac{15}{16}$ and so from the previous line,

$$\cos x = -\sqrt{\frac{15}{16}} = -\frac{\sqrt{15}}{4}.$$

Thus:

$$\sin 2x = 2\sin x \cos x = 2 \cdot \frac{1}{4} \cdot -\frac{\sqrt{15}}{4} = \frac{-\sqrt{15}}{8}$$

$$\cos 2x = \cos^2 x - \sin^2 x = \frac{15}{16} - \left(\frac{1}{4}\right)^2 = \frac{14}{16} = \frac{7}{8}$$

$$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{-\sqrt{15}/8}{7/8} = \frac{-\sqrt{15}}{7}$$

11.
$$\cos^4 x \sin^4 x = (\cos^2 x \sin^2 x)^2$$

$$= \left(\frac{1+\cos 2x}{2} \cdot \frac{1-\cos 2x}{2}\right)^2$$

$$= \left(\frac{1-\cos^2 2x}{4}\right)^2$$

$$= \left(\frac{\sin^2 2x}{4}\right)^2$$

$$= \left(\frac{1}{4} \cdot \frac{1-\cos(2 \cdot 2x)}{2}\right)^2$$

$$= \left(\frac{1-\cos 4x}{8}\right)^2$$

$$= \frac{1}{64}(1-\cos 4x)^2$$

$$= \frac{1}{64}(1 - 2\cos 4x + \cos^2 4x)$$

$$= \frac{1}{64}\left(1 - 2\cos 4x + \frac{1+\cos(2 \cdot 4x)}{2}\right)$$

$$= \frac{1}{64} - \frac{1}{32}\cos 4x + \frac{1+\cos 8x}{128}$$

$$= \frac{1}{64} - \frac{\cos 4x}{32} + \frac{1}{128} + \frac{\cos 8x}{128}$$

$$= \frac{3}{128} - \frac{\cos 4x}{32} + \frac{\cos 8x}{128}$$

11 (again) Another way to 11 is as follows.
We can use the identity $\sin 2x = 2\sin x\cos x$ to get
$$\tfrac{1}{2}\sin 2x = \sin x\cos x.$$

Thus
$$\begin{aligned}\sin^4 x\cos^4 x &= (\sin x\cos x)^4\\ &= \left(\tfrac{1}{2}\sin 2x\right)^4\\ &= \tfrac{1}{16}\sin^4 2x\\ &= \tfrac{1}{16}\left(\frac{1-\cos 4x}{2}\right)^2\\ &= \frac{1}{16}\cdot\frac{(1-\cos 4x)^2}{4}\\ &= \frac{1-2\cos 4x+\cos^2 4x}{64}\\ &= \frac{1}{64}-\frac{\cos 4x}{32}+\frac{1+\cos 8x}{128}\\ &= \frac{3}{128}-\frac{\cos 4x}{32}+\frac{\cos 8x}{128}\end{aligned}$$

Same answer, different route.

16. $\tan 15° = \tan\frac{30°}{2} = \dfrac{1-\cos 30°}{\sin 30°} = \dfrac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2\left(1-\frac{\sqrt{3}}{2}\right) = 2-\sqrt{3}$

23. a) $2\sin 18°\cos 18° = \sin(2\cdot 18°) = \sin 36°$
b) $2\sin 3\theta\cos 3\theta = \sin(2\cdot 3\theta) = \sin 6\theta.$

These are the double-angle formulas from right to left.

28. a) $\sqrt{\dfrac{1-\cos 30°}{2}} = \sin\left(\frac{30°}{2}\right) = \sin 15°$
b) $\sqrt{\dfrac{1-\cos 8\theta}{2}} = \sin\left(\frac{8\theta}{2}\right) = \sin 4\theta$

These are the half-angle formulas