

7.2 3, 6, 13, 17, 21, 29, 34, 49

$$\begin{aligned} 3. \sin 165^\circ &= \sin(45^\circ + 120^\circ) \\ &= \sin 45^\circ \cos 120^\circ + \cos 45^\circ \sin 120^\circ \\ &= \frac{\sqrt{2}}{2} \cdot \frac{-1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} \\ &= \frac{-\sqrt{2} + \sqrt{6}}{4} \end{aligned}$$

$$\begin{aligned} 6. \cos \frac{11\pi}{12} &= \cos\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right) = \cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) \\ &= \cos \frac{2\pi}{3} \cos \frac{\pi}{4} - \sin \frac{2\pi}{3} \sin \frac{\pi}{4} \\ &= \frac{-1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{-\sqrt{2} - \sqrt{6}}{4} \end{aligned}$$

$$\begin{aligned} 13. \cos \frac{3\pi}{7} \cos \frac{2\pi}{21} + \sin \frac{3\pi}{7} \sin \frac{2\pi}{21} &= \cos\left(\frac{3\pi}{7} - \frac{2\pi}{21}\right) \\ &= \cos\left(\frac{9\pi}{21} - \frac{2\pi}{21}\right) = \cos\left(\frac{7\pi}{21}\right) \\ &= \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \end{aligned}$$

$$17. \tan\left(\frac{\pi}{2} - u\right) = \cot u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \frac{\tan \frac{\pi}{2} - \tan u}{1 + \tan \frac{\pi}{2} \tan u}$$

but here is a problem since $\tan \frac{\pi}{2}$ is undefined

To get around this, we employ the substitution $\tan\left(\frac{\pi}{2} - u\right) = \frac{\sin\left(\frac{\pi}{2} - u\right)}{\cos\left(\frac{\pi}{2} - u\right)}$.

$$\begin{aligned} \text{LHS} &= \frac{\sin\left(\frac{\pi}{2} - u\right)}{\cos\left(\frac{\pi}{2} - u\right)} = \frac{\sin \frac{\pi}{2} \cos u - \cos \frac{\pi}{2} \sin u}{\cos \frac{\pi}{2} \cos u + \sin \frac{\pi}{2} \sin u} \\ &= \frac{1 \cdot \cos u - 0 \cdot \sin u}{0 \cdot \cos u + 1 \cdot \sin u} \\ &= \frac{\cos u}{\sin u} = \cot u = \text{RHS.} \end{aligned}$$

$$21. \sin\left(x - \frac{\pi}{2}\right) = -\cos x$$

$$\begin{aligned} \text{LHS} &= \sin\left(x - \frac{\pi}{2}\right) = \sin x \cos \frac{\pi}{2} - \cos x \sin \frac{\pi}{2} \\ &= \sin x \cdot 0 - \cos x \cdot 1 \\ &= -\cos x = \text{RHS} \end{aligned}$$

$$29. \sin(x+y) - \sin(x-y) = 2\cos x \sin y.$$

$$\text{LHS} = \sin(x+y) - \sin(x-y)$$

$$= (\sin x \cos y + \cos x \sin y) - (\sin x \cos y - \cos x \sin y)$$

$$= \cos x \sin y - -\cos x \sin y$$

[The first and third terms cancel]

$$= 2\cos x \sin y = \text{RHS}$$

$$34. 1 - \tan x \tan y = \frac{\cos(x+y)}{\cos x \cos y}$$

$$\text{RHS} = \frac{\cos(x+y)}{\cos x \cos y} = \frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}$$

$$= \frac{\cos x \cos y}{\cos x \cos y} - \frac{\sin x}{\cos x} \cdot \frac{\sin y}{\cos y}$$

$$= 1 - \tan x \tan y = \text{LHS}$$

$$49. y = \sin^2(x + \frac{\pi}{4}) + \sin^2(x - \frac{\pi}{4})$$

$$Y_1 = \sin^2(x + \pi/4) + \sin^2(x - \pi/4)$$

(Be sure to be in radian mode when graphing)

a) The graph suggests that $\sin^2(x + \frac{\pi}{4}) + \sin^2(x - \frac{\pi}{4}) = 1$.

b) $\text{LHS} = \sin^2(x + \frac{\pi}{4}) + \sin^2(x - \frac{\pi}{4})$

$$= (\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4})^2 + (\sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4})^2$$

$$= (\frac{\sqrt{2}}{2}(\sin x + \cos x))^2 + (\frac{\sqrt{2}}{2}(\sin x - \cos x))^2$$

$$= \frac{1}{2}(\sin x + \cos x)^2 + \frac{1}{2}(\sin x - \cos x)^2$$

$$= \frac{1}{2}(\sin^2 x + 2\sin x \cos x + \cos^2 x) + \frac{1}{2}(\sin^2 x - 2\sin x \cos x + \cos^2 x)$$

$$= \frac{1}{2}(2\sin^2 x + 2\cos^2 x)$$

$$= \sin^2 x + \cos^2 x$$

$$= 1 = \text{RHS}$$

END OF 7.2