7.1 1, 5, 11, 15, 16, 43, 49, 52, 63, 85, 91, 94

1. $\sin t \cot t = \sin t \cdot \frac{\cos t}{\sin t}$

$= \cos t.$

5. $\tan^2 x - \sec^2 x = \left(\frac{\sin x}{\cos x}\right)^2 - \left(\frac{1}{\cos x}\right)^2$

$= \frac{\sin^2 x - 1}{\cos^2 x}$

$= \frac{-\cos^2 x}{\cos^2 x}$ [By Pythagorean Identity]

$= -1.$

11. $\frac{1+\cos y}{1+\sec y} = \frac{1+\cos y}{1+\frac{1}{\cos y}} \cdot \frac{\cos y}{\cos y}$

$= \frac{\cos y (1+\cos y)}{(\cos y + 1)}$

$= \cos y.$

Tip: Sometimes you can quickly see what needs to be done to simplify, sometimes you can't. My first attempt at this problem took me nine steps to get to $\cos y$. Then I came up with a quicker way, shown above. If your way differs from mine but all your steps are correct, you're still giving a correct simplification.

15. $$\frac{1+\csc x}{\cos x+\cot x} = \frac{1+\frac{1}{\sin x}}{\cos x+\frac{\cos x}{\sin x}} = \frac{\left(1+\frac{1}{\sin x}\right)}{\cos x\left(1+\frac{1}{\sin x}\right)}$$

$$= \frac{1}{\cos x} = \sec x.$$

16. $$\frac{\sin x}{\csc x}+\frac{\cos x}{\sec x} = \frac{\sin x}{\frac{1}{\sin x}}+\frac{\cos x}{\frac{1}{\cos x}}$$

$$= \frac{\sin x}{\frac{1}{\sin x}}\cdot\frac{\sin x}{\sin x}+\frac{\cos x}{\frac{1}{\cos x}}\cdot\frac{\cos x}{\cos x}$$

$$= \sin^2 x+\cos^2 x$$

$$= 1.$$

43. $(\cot x-\csc x)(\cos x+1) = -\sin x$

LHS $= (\cot x-\csc x)(\cos x+1) = \cot x\cos x+\cot x-\csc x\cos x-\csc x$

$$= \frac{\cos x}{\sin x}\cos x+\frac{\cos x}{\sin x}-\frac{1}{\sin x}\cos x-\frac{1}{\sin x}$$

$$= \frac{\cos^2 x}{\sin x}-\frac{1}{\sin x}$$ [middle terms canceled out]

$$= \frac{1}{\sin x}(\cos^2 x-1)$$

$$= \frac{1}{\sin x}(-\sin^2 x)$$ [Pyth. identity]

$$= -\sin x = \text{RHS}$$

REMEMBER: When proving identities ONLY WORK WITH ONE SIDE AT A TIME! Start with one side and try to simplify it to the other, or simplify each side independently to the same thing.