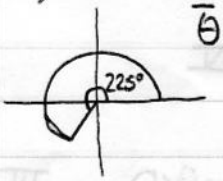
6.3

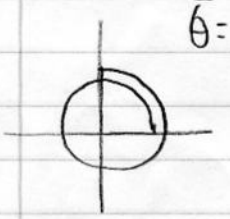
2. Find the reference angle.

a) 225°



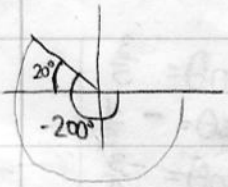

$\bar{\theta} = 225° - 180° = 45°$

b) 450°



$\bar{\theta} = 450° - 360° = 90°$

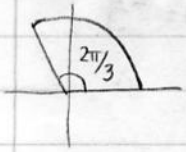
c) -200°




$\bar{\theta} = |-200° + 180°| = 20°$

19, 21, 22 Find exact value.

19. $\sin\frac{2\pi}{3}$

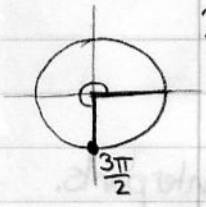



Reference angle: $\pi - \frac{2\pi}{3} = \frac{\pi}{3}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Since $\frac{2\pi}{3}$ has terminal point in quadrant II, $\sin\frac{2\pi}{3} = +\sqrt{3}/2$

21. $\sin\frac{3\pi}{2}$




Reference angle: $\frac{3\pi}{2} - \pi = \frac{\pi}{2}$

$\sin\frac{\pi}{2} = +1$.

Since $\frac{3\pi}{2}$ is in quadrant III (and IV), $\sin\frac{3\pi}{2} = -1$

22. $\cos\frac{7\pi}{3}$

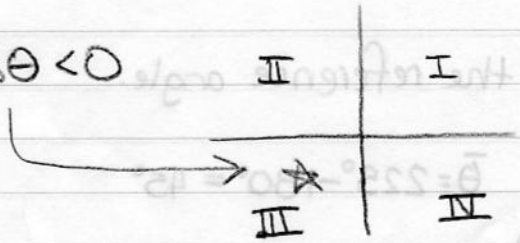
Reference angle: $\frac{7\pi}{3} - 2\pi = \frac{\pi}{3}$

Notice that $\frac{7\pi}{3}$ and $\frac{\pi}{3}$ are coterminal so

$\cos\frac{7\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$.

31. $\sin\theta<0$ and $\cos\theta<0$

Quadrant III

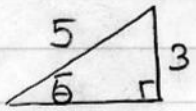



32. $\tan\theta<0$ and $\sin\theta<0$

Recall $\tan\theta<0$ in quadrants II and IV, and $\sin\theta<0$ in III and IV, so $\theta$ must have terminal point in IV.

41. $\sin\theta=\frac{3}{5}$, $\theta$ in quadrant II

We first sketch the triangle for $\bar{\theta}$




4 ← (use the Pythagorean theorem)

From this we get

$\sin\bar{\theta}=\frac{3}{5}$
$\cos\bar{\theta}=\frac{4}{5}$
$\tan\bar{\theta}=\frac{3}{4}$
$\csc\bar{\theta}=\frac{5}{3}$
$\sec\bar{\theta}=\frac{5}{4}$
$\cot\bar{\theta}=\frac{4}{3}$

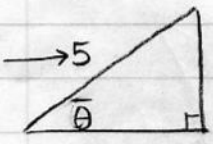
and so since we are in quadrant II:

$\sin\theta=\frac{3}{5}$
$\cos\theta=-\frac{4}{5}$
$\tan\theta=-\frac{3}{4}$
$\csc\theta=\frac{5}{3}$
$\sec\theta=-\frac{5}{4}$
$\cot\theta=-\frac{4}{3}$

43. $\tan\theta=-\frac{3}{4}$, $\cos\theta>0$

Again, draw the triangle for $\bar{\theta}$

(Pyth. thm.) →




Since $\tan\theta$ is negative and $\cos\theta$ is positive, then $\sin\theta$ must be negative. The signs for $\cot\theta$, $\csc\theta$, and $\sec\theta$ follow from their counterparts.

From the triangle:

$\sin\bar{\theta}=\frac{3}{5}$
$\cos\bar{\theta}=\frac{4}{5}$
$\tan\bar{\theta}=\frac{3}{4}$
$\csc\bar{\theta}=\frac{5}{3}$
$\sec\bar{\theta}=\frac{5}{4}$
$\cot\bar{\theta}=\frac{4}{3}$

applying the proper signs:

$\sin\theta=-\frac{3}{5}$
$\cos\theta=\frac{4}{5}$
$\tan\theta=-\frac{3}{4}$
$\csc\theta=-\frac{5}{3}$
$\sec\theta=\frac{5}{4}$
$\cot\theta=-\frac{4}{3}$