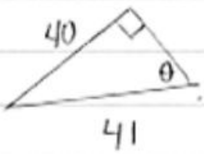
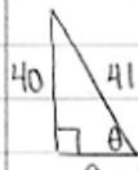
6.2 Trigonometry of Right Triangles

3. Find the six trigonometric ratios for the given triangle



I like to draw my right triangles with the right angle in the lower left and marked angle θ in the lower right.



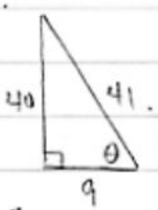
We can use the Pythagorean theorem to find the length of a, the remaining side.

$a^2 + 40^2 = 41^2$

$a^2 = 41^2 - 40^2 = 81$

$a = \sqrt{81} = 9.$

Thus we get

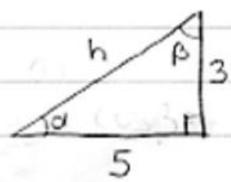


so we get:

$$\left\{ \begin{array}{|ll|} \sin\theta = \frac{40}{41} & \csc\theta = \frac{41}{40} \\ \cos\theta = \frac{9}{41} & \sec\theta = \frac{41}{9} \\ \tan\theta = \frac{40}{9} & \cot\theta = \frac{9}{40} \end{array} \right.$$

(see the chart on p. 483 if you don't know where these numbers came from)

7.



We can find the length of h by the Pyth. theorem:

$h^2 = 3^2 + 5^2 = 34$

$h = \sqrt{34}.$

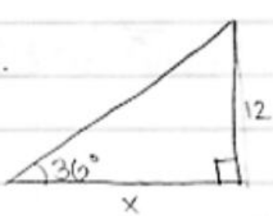
Thus we get

a) $\sin\alpha = \frac{3}{h} = \frac{3}{\sqrt{34}}$ b) $\tan\alpha = \frac{3}{5}$ c) $\sec\alpha = \frac{h}{5} = \frac{\sqrt{34}}{5}$

$\cos\beta = \frac{3}{h} = \frac{3}{\sqrt{34}}$ $\cot\beta = \frac{3}{5}$ $\csc\beta = \frac{h}{5} = \frac{\sqrt{34}}{5}$

[Notice that each pair is the same. The "co" of cosine, cotangent, and cosecant is short for "complement". α and β are "complementary" angles because $\alpha + \beta = 90°$. $\cos\beta = \sin\alpha$ since β is the complement of α.]

13.



Find the side labelled x.

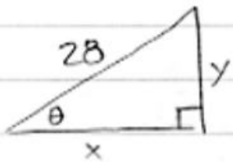
Since we are given the adjacent and opposite sides for 36°, let's use tan 36° (we could just as easily use cot 36°).

$\tan 36° = \frac{12}{x}$

$x \tan 36° = 12$

$x = \frac{12}{\tan 36°} \approx$ **16.51658**

15. Express x and y in terms of trig. ratios of θ.

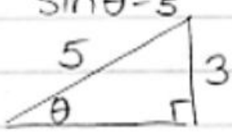


$\cos\theta = \frac{x}{28}$, so $x = 28\cos\theta$

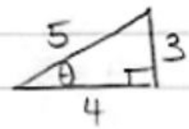
$\sin\theta = \frac{y}{28}$, so $y = 28\sin\theta$

17. Sketch a tringle with acute θ and find the 5 other trig. ratios of θ.

$\sin\theta = \frac{3}{5}$



By Pyth. theorem, we know the remaining side has length 4.



Thus we get:

$\sin\theta = \frac{3}{5}$ (given)

$\cos\theta = \frac{4}{5}$

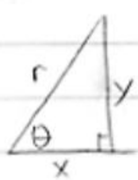
$\tan\theta = \frac{3}{4}$

$\csc\theta = 5/3$

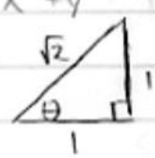
$\sec\theta = 5/4$

$\cot\theta = 4/3$.

19. $\cot\theta = 1$. This means $x = y$ in the triangle below, so letting $x = y = 1$ is easiest.



Then $r^2 = x^2 + y^2$, so $r = \sqrt{1^2+1^2} = \sqrt{2}$.



$\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

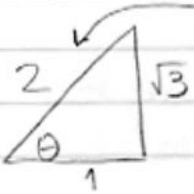
$\tan\theta = \frac{1}{1} = 1$

$\csc\theta = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\sec\theta = \frac{\sqrt{2}}{1} = \sqrt{2}$

20. $\tan\theta = \sqrt{3}$. Since $\tan\theta = \frac{\sqrt{3}}{1}$, let $y = \sqrt{3}$, $x = 1$.

By Pyth thm, the hypoteneuse has length $\sqrt{(\sqrt{3})^2 + 1^2} = 2$.



$\sin\theta = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{1}{2}$

$\tan\theta = \sqrt{3}$ (given)

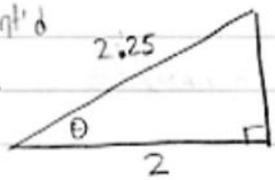
$\csc\theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec\theta = \frac{2}{1} = 2$

$\cot\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

6.2 cont'd

33.

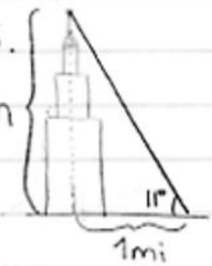


$\sin\theta = \frac{1}{2.25} = 0.44$ $\quad\csc\theta = 2.25$

$\cos\theta = \frac{2}{2.25} = 0.89$ $\quad\sec\theta = \frac{2.25}{2} = 1.125$

$\tan\theta = \frac{1}{2} = 0.5$ $\quad\cot\theta = \frac{2}{1} = 2$

35.



Find the height of the Empire State Building.

Once you've drawn the picture, this is just like problem 13.

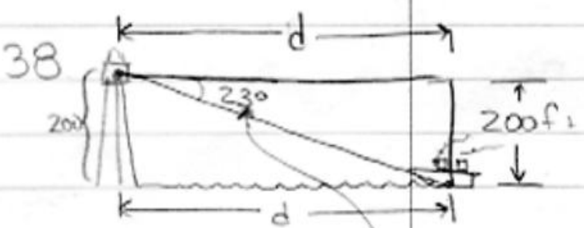
$\tan 11° = \frac{h}{1}$

So $h = \tan 11° = 0.194$ mi

Normally buildings are not measured in miles.

$0.194\text{mi} \cdot \frac{5280\text{ft}}{1\text{mi}} = 1026.3$ ft.

38



(This is the angle of depression)

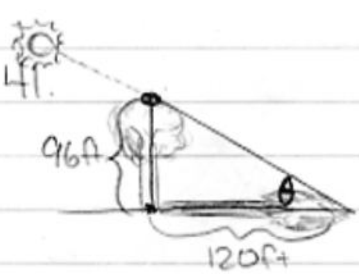
Find the distance from the lighthouse to the ship

Again, once you've drawn the picture its just like problem 13.

$\tan 23° = \frac{200}{d}$

$d = \frac{200}{\tan 23°} = 471.17$ ft

41.



Now we have that $\tan\theta = \frac{96}{120}$.

To find θ we'll need the calculator's $\boxed{\text{TAN}^{-1}}$ key.

$\theta = \tan^{-1}\left(\frac{96}{120}\right)$

$= 38.66°$ if you're in Degree Mode

($= 0.6747$ if you're in Radian Mode)

Those sorts of problems usually use degrees.