6.1 Angle Measures

1. $36^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{5}$

4. $-72^\circ \cdot \frac{\pi}{180^\circ} = \frac{-2\pi}{5}$

5. $60^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{3}$

12. $2 \cdot \frac{180^\circ}{\pi} = \frac{360^\circ}{\pi} \approx 114.6^\circ$

14. $\frac{2\pi}{9} \cdot \frac{180^\circ}{\pi} = 40^\circ$

15. $\frac{-\pi}{12} \cdot \frac{180^\circ}{\pi} = -15^\circ$

18, 19 Find two positive angles and two negative angles coterminal with the given angle.

18. 135°.

$$\left.\begin{aligned} 135^\circ + 360^\circ &= 495^\circ \\ 135^\circ + 360^\circ + 360^\circ &= 855^\circ \\ 135^\circ - 360^\circ &= -225^\circ \\ 135^\circ - 360^\circ - 360^\circ &= -585^\circ \end{aligned}\right\} \text{all are coterminal with } 135^\circ.$$

19. $\frac{3\pi}{4}$

$$\left.\begin{aligned} \frac{3\pi}{4} + 2\pi &= \frac{11\pi}{4} \\ \frac{3\pi}{4} + 2\pi + 2\pi &= \frac{19\pi}{4} \\ \frac{3\pi}{4} - 2\pi &= -\frac{5\pi}{4} \\ \frac{3\pi}{4} - 2\pi - 2\pi &= \frac{-13\pi}{4} \end{aligned}\right\} \text{all are coterminal with } \frac{3\pi}{4}.$$

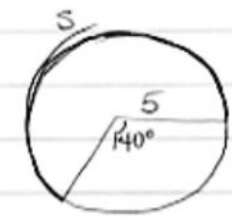
23, 25 Are the given angles coterminal?

23. $70^\circ, 430^\circ$. $\quad 430^\circ - 360^\circ = 70^\circ$; so 70° and 340° are coterminal

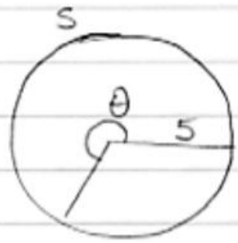
25. $\frac{5\pi}{6}, \frac{17\pi}{6}$ $\quad \frac{17\pi}{6} - 2\pi = \frac{5\pi}{6}$, so $\frac{5\pi}{6}$ and $\frac{17\pi}{6}$ are coterminal.

35. $\frac{17\pi}{6} - 2\pi = \frac{5\pi}{6}$. Since $0 \le \frac{5\pi}{6} \le 2\pi$, $\frac{5\pi}{6}$ is the desired angle.

41. Find the length of arc s.



We can use the arclength formula on p.477, but we need to know the angle which s subtends in radians. That is we need to know θ in radians



First 140° is

$140° \cdot \frac{\pi}{180} = \frac{7\pi}{9}$ radians.

That means that $\theta = 2\pi - \frac{7\pi}{9} = \frac{11\pi}{9}$ radians.

Hence $s = r\theta = 5 \cdot \frac{11\pi}{9} = \frac{55\pi}{9} \approx 19.2$

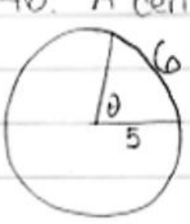
You could have alternately Calculated θ in degrees and then converted θ to radians to get the same answer

44. Find length of arc that subtends a central angle of 45° in a circle of radius 10m.

$45° \cdot \frac{\pi}{180°} = \frac{\pi}{4}$ radians

$s = 10 \cdot \frac{\pi}{4} = \frac{5\pi}{2} \approx 7.85$ m

46. A central angle of radius 5 is subtended by an arc of length 6m.



Find θ in radians and degrees.

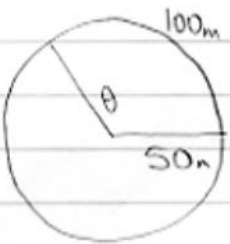
By the arclength formula, $s = r\theta$,

$6 = 5\theta$

So $\theta = \frac{5}{6}$ radians

Thus $\theta = \frac{5}{6} \cdot \frac{180°}{\pi} = \frac{216°}{\pi} \approx 68.75°$

47. An arc of length 100m subtends an angle θ on a circle of radius 50m



Find θ in radians and degrees.

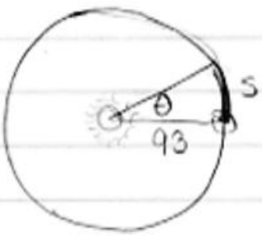
Again, use the formula $s = r\theta$, or rather $\theta = \frac{s}{r}$.

$\theta = \frac{100}{50} = 2$ radians

So $\theta = 2 \cdot \frac{180°}{\pi} = \frac{360°}{\pi} \approx 114.59°$

6.1 cont'd

53. Find the distance the earth travels in one day in its path around the sun.



Let's measure in millions of miles, so $r = 93$.

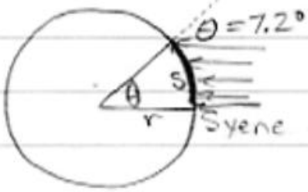
Now let's figure out θ.

$\theta = \frac{1}{365} \cdot 2\pi = \frac{2\pi}{365}$

Thus $s = r\theta = 93 \cdot \frac{2\pi}{365} \approx 1.600921$ million miles

$= 1600921$ miles

54. Figure out the radius and circumference of the earth based on Eratosthenes' observations.



First notice that the central angle θ is the same as the deviation from the zenith. Thus $\theta = 7.2°$, but we'll need this in radians

$\theta = 7.2° \cdot \frac{\pi}{180°} = \frac{\pi}{25}$

The problem says $s = 500$ mi, so we can use the arc length formula, $s = r\theta$.

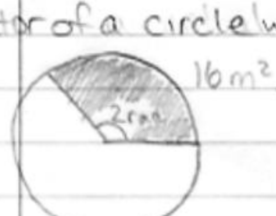
$500 = r \cdot \frac{\pi}{25}$, so

$r = \frac{500 \cdot 25}{\pi} \approx 3978.87$ mi.

The circumference $C = 2\pi r$, so

$C = 2\pi \cdot \frac{500 \cdot 25}{\pi} = 25000$ mi.

59. The area of a sector of a circle with a central angle of 2 radian is $16 m^2$. Find the radius.



Use the sector area formula from p.478 $A = \frac{1}{2} r^2 \theta$. Solve for r to get $\frac{2A}{\theta} = r^2$

$r = \sqrt{\frac{2A}{\theta}}$.

Thus $r = \sqrt{\frac{2 \cdot 16}{2}} = \sqrt{16} = 4$ m.

END of 6.1