4.5

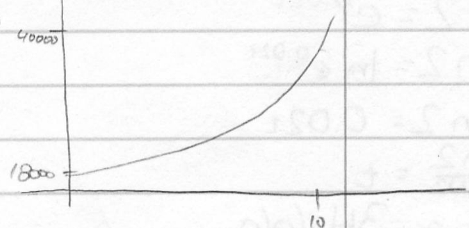
3. A fox population has a relative growth rate of 8% per year. In 2000 the population was 18000.

a) Find a function that models the population t years after 2000. Here we use the formula on page 376, with $n_0 = 18000$, $r = 0.08$, and time t. Then $n(t) = 18000e^{0.08t}$.

b) Find the population in 2008 by the function in part a.

Now $t = 8$, so $n(8) = 18000e^{0.08 \cdot 8} = 34136.66$

c)



8. A culture of 1500 bacteria doubles every 30 minutes.

a) Find a function that models the number of bacteria after t minutes.

To use the formula on page 376, we need to know the relative growth rate r. We can use the doubling time to figure r out. We know at $t = 30$ that the population $n(30) = 3000$ (twice 1500).

Thus $3000 = 1500e^{r \cdot 30}$, and so we solve for r.

$$2 = e^{30r}$$

$$\ln 2 = 30r$$

$$r = \frac{\ln 2}{30} \approx 0.0231.$$

Now we make t generic again and plug in r:

$n(t) = 1500e^{0.0231t}$ models the population after t minutes.

b) Find the number of bacteria after 2 hours.

This is simply $n(2)$. $n(2) = 1500e^{0.0231 \cdot 2} = 1570.93 \approx 1571$ bacteria.

c) How long until there are 4000 bacteria?

We must solve $4000 = 1500e^{0.0231 \cdot t}$ for t.

$$\frac{4000}{1500} = e^{0.0231t}$$

$$\ln(2.6667) = 0.0231t$$

$$t = \frac{\ln(2.6667)}{0.0231} \approx 42.46 \text{ minutes}$$

11. The world population in 1995 was 5.7 billion, and the relative growth rate was 2% per year.

a) By what year will the population have doubled.

We want to know in what year will the population be $2 \cdot 5.7 = 11.4$ (billion)

Thus letting 1995 be $t=0$ we get the formula

$$n(t) = 5.7e^{0.02t}$$

For part a, we must solve the equation

$$11.4 = 5.7e^{0.02t}$$
$$2 = e^{0.02t}$$
$$\ln 2 = \ln e^{0.02t}$$
$$\ln 2 = 0.02t$$
$$\frac{\ln 2}{0.02} = t$$
$$t \approx 34.66 \text{ years.}$$

Hence 35 years after 1995, that is in the year 2030 the population will have doubled.

b) By what year will the population have tripled?

Now the target population is $3 \cdot 5.7 = 17.1$, so we solve

$$17.1 = 5.7e^{0.02t}$$
$$3 = e^{0.02t}$$
$$\ln 3 = 0.02t$$
$$t = \frac{\ln 3}{0.02} \approx 54.93$$

So in 55 years after 1995, the year 2050, the population will have tripled.

12. In 1950, $n_0 = 10586223$ and in 1980 $n(30) = 23\,668\,562$.

a) Find $n(t)$.

Here we must find r again based on what we are given. Solve for r in

$$23668562 = 10586223e^{r \cdot 30}$$

to get $r = \dfrac{\ln\left(\frac{23668562}{10586223}\right)}{30} \approx 0.312$ (Yes, I skipped some steps)

Thus $n(t) = 10586223e^{0.312t}$.

b) Find the doubling time.

We could solve for t as in part a of question 11. There is an easier way, though. Now we just solve for t in the equation $2 = e^{0.312t}$. This skips the step of finding the target population and messing with those ugly long numbers.

Here you should get $t = \dfrac{\ln 2}{0.312} = 2.22$ years

c) Use Google.