

4.4 cont'd

68. The learning curve is given by the function $P(t) = M - Ce^{-kt}$.

a) Express the learning time as a function of P .

We want to solve $P = M - Ce^{-kt}$ for t .

$$P = M - Ce^{-kt}$$

$$P - M = -Ce^{-kt}$$

$$\frac{P-M}{-C} = e^{-kt}$$

$$\frac{M-P}{C} = e^{-kt}$$

$$\ln\left(\frac{M-P}{C}\right) = \ln e^{-kt}$$

$$\ln\left(\frac{M-P}{C}\right) = -kt$$

$$t = -\frac{1}{k} \ln\left(\frac{M-P}{C}\right)$$

$$= -\frac{1}{k} \ln\left(\frac{M-P}{C}\right)$$

$$= \ln\left(\frac{M-P}{C}\right)^{-\frac{1}{k}}$$

Thus $t(P) = \ln\left(\frac{M-P}{C}\right)^{-\frac{1}{k}}$
 "t of P" ↗

Now we have moved everything not involving t to the left and have all the t 's on the right.

This is good enough, but we could simplify.

Since $\frac{P-M}{-C} = \frac{M-P}{C}$.

b) Use the formula from part a using $M=20$, $C=14$, $k=0.024$, $P=12$.

You should get $t = 23.32$ months.

69, 76 Use the graphing calculator to find the intersections of the two graphs. Be sure you find all of them by zooming out far enough. For 76 you should get $x = -0.8856$ and $x = 0.7059$.

79. $2 < 10^x < 5$

$$\log 2 < \log 10^x < \log 5$$

$$\log 2 < x \log 10 < \log 5$$

$$\log 2 < x < \log 5$$

Applying log does not change the direction of the inequalities.

80. $x^2 e^x - 2e^x < 0$

$$(x^2 - 2)e^x < 0$$

Use a sign chart and check certain values.

$x^2 - 2 = 0$ when $x = \pm\sqrt{2}$. $e^x > 0$ for all x , so $e^x \neq 0$ for any x .

+	o	-	o	+
$-\sqrt{2}$			$\sqrt{2}$	

Therefore $x^2 e^x - 2e^x < 0$ for the interval $(-\sqrt{2}, \sqrt{2})$

4.4

$$50. \ln(x-1) + \ln(x+2) = 1$$

$$\ln((x-1)(x+2)) = 1$$

$$e^{\ln((x-1)(x+2))} = e^1$$

$$(x-1)(x+2) = e$$

$$x^2 + x - 2 - e = 0$$

$$x^2 + x - (e+2) = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-e+2)}}{2}$$

$$= \frac{-1 \pm \sqrt{1 + 4e + 8}}{2}$$

$$= \frac{-1 \pm \sqrt{9 + 4e}}{2}$$

$x \approx 1.729$ and $x \approx -2.729$
are the solutions

$$52. (\log x)^3 = 3 \log x$$

$$(\log x)^3 - 3(\log x) = 0$$

$$\log x ((\log x)^2 - 3) = 0 \quad \text{Factor out a } \log x.$$

So $\log x = 0$ or $(\log x)^2 - 3 = 0$

$\log x = 0$ means $x = 1$

$$(\log x)^2 - 3 = 0 \text{ means } (\log x)^2 = 3$$

$$\log x = \pm \sqrt{3}$$

$$10^{\log x} = 10^{\pm \sqrt{3}}$$

$$x = 10^{-\sqrt{3}} \text{ or } 10^{\sqrt{3}}$$

Thus $x = 1$, $10^{-\sqrt{3}}$, and $10^{\sqrt{3}}$ are the three values
for which $(\log x)^3 = 3 \log x$ is true.